भारतीय ज्ञान परंपरा

संस्कृति, दर्शन, शैक्षिक विचारधाराएँ और नवाचार

सम्पादक :
डॉ. दिनेश चन्द्र शर्मा

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रचना - **भारतीय ज्ञान परम्परा में शिक्षा के समाजशास्त्रीय आधार**

सम्पादक - **डॉ. दिनेश चन्द्र शर्मा**

मुद्रक - **श्रीनिवास प्रिन्टोग्राफिक्स, अजमेर**

6. Discrete Mathematics

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Discrete Mathematics is a branch of mathematics that is concerned with "discrete" mathematical structures instead of "continuous". Discrete mathematical structures include objects with distinct values like graphs, integers, logic-based statements, etc. we have covered all the topics of Discrete Mathematics for computer science like set_theory, recurrence relation, group theory, and graph theory.

Set Theory - The concept of Set Theory was propounded in the year 1874 by Georg Cantor in his paper name 'On a Property of Collection of All Real Algebraic Numbers'.

Sets are defined as "a well-defined collection of objects". Let's say we have a set of Natural Numbers then it will have all the natural numbers as its member and the collection of the numbers is well defined that they are natural numbers. For Example :- A set of Natural Numbers is given by N = {1, 2, 3, 4.....}.

Elements of a Set

The objects contained by a set are called the elements of the set. For Example: In the set of Natural Numbers 1, 2, 3, etc. are the objects contained by the set, hence they are called the elements of the set of Natural Numbers. We can also say that 1 belongs to set N. It is represented as $1 \in N$, where $\in$ is the symbol of belongs to.

Cardinal Number of a Set

The number of elements present in a set is called the Cardinal Number of a Set. For Example - Let's say P is a set of the first five prime numbers given by P = {2, 3, 5, 7, 11}, then the Cardinal Number of set P is 5. The Cardinal Number of Set P is represented by n(P) or $|P| = 5$.

Some Standard Sets used in Set Theory

Set of Natural Numbers is denoted by N

Set of Whole Numbers is denoted by W

Set of Integers is denoted by Z

Set of Rational Numbers is denoted by Q

Set of Irrational Numbers is denoted by P

Set of Real Numbers is denoted by R

Representation of Set

Sets are primarily represented in two forms.

* Roster Form
* Set Builder Form

1. Roster Form

In the Roster Form of the set, the elements are placed inside braces {} and are separated by commas.

Let's say we have a set of the first five prime numbers then it will be represented by P = {2, 3, 5, 7, 11}. Here the set P is an example of a finite set as the number of elements is finite, however, we can come across a set that has infinite elements then in that case the roster form is represented in the manner that some elements are placed followed by dots to represent infinity inside the braces.

Let's say we have to represent a set of Natural Numbers in Roster Form then its Roster Form is given as N = {1, 2, 3, 4.....}.

Set does not contain duplicate elements. For Example:

If A represents a set that contains all letter of word TREE, then correct roster form representation will be:

A = {T,R,E}

A ≠ {T,R,E,E}

2. Set Builder Form

In Set Builder Form, a rule or a statement describing the common characteristics of all the elements is written instead of writing the elements directly inside the braces. For Example: A set of all the prime numbers less than or equal to 10 is given as P = {p : p is a prime number ≤ 10}. In another example, the set of Natural Numbers in set builder form is given as N = {nn is a natural number}.

Set Theory Symbols

Symbol	**Explanation**
{}	Set

$X \in A$	X is an element of set A
$X \notin A$	X is not an element of set A
$\exists$ or $\exists$	There exist or there doesn't exist
Φ	Empty Set
$A = B$	Equal Sets
n (A)	Cardinal Number of Set A
P (A)	Power Set
$A \subseteq B$	A is a subset of B
$A \subset B$	A is the Proper subset of B
$A \not\subseteq B$	A is not a subset of B
$B \supseteq A$	B is the superset of A
$B \supset A$	B is a proper superset of A
$B \supseteq A$	B is not a superset of A
$A \cup B$	A union B
$A \cap B$	A intersection B
A'	Complement of Set A

Types of Sets

Sets can be classified into many categories. Some of which are finite, infinite, subset, universal, proper, power, singleton set, etc.

(1) Finite Sets : A set is said to be finite if it contains exactly n distinct element where n is a non-negative integer. Here, n is said to be "cardinality of sets." The cardinality of sets is denoted by $|A|$, # A, card (A) or n (A).

Example:

1. Cardinality of empty set 0 is 0 and is denoted by $|0| = 0$
2. Sets of even positive integer is not a finite set.

A set is called a finite set if there is one to one correspondence between the elements in the set and the element in some set n, where n is a natural number and n is the cardinality of the set. Finite Sets are also called numerable sets. n is termed as the cardinality of sets or a cardinal number of sets.

(2) Infinite Sets :- A set which is not finite is called as Infinite Sets.

Countable Infinite :- If there is one to one correspondence between the elements in set and element in N. A countable infinite set is also known as Denumerable. A set that is either finite or denumerable is known as countable. A set which is not countable is known as Uncountable. The set of a non-negative even integer is countable Infinite.

Uncountable Infinite :- A set which is not countable is called Uncountable Infinite Set or non- denumerable set or simply Uncountable.

Example :- Set R of all +ve real numbers less than 1 that can be represented by the decimal form 0. a_1, a_2, a_3 ..... Where a_1 is an integer such that $0 \quad a_1 \leq a \leq 9$.

(3) Subsets :- If every element in a set A is also an element of a set B, then A is called a subset of B. It can be denoted as $A \subseteq B$. Here B is called Superset of A.

Example :- If A= {1, 2} and B= {4, 2, 1} the A is the subset of B or $A \subseteq B$.

Properties of Subsets :-

1. Every set is a subset of itself.
2. The Null Set i.e.Ø is a subset of every set.
3. If A is a subset of B and B is a subset of C, then A will be the subset of C. If $A \subset B$ and $B \subset C \Rightarrow A \subset C$
4. A finite set having n elements has 2^n subsets.

(4) Proper Subset :- If A is a subset of B and $A \neq B$ then A is said to be a proper subset of B. If A is a proper subset of B then B is not a subset of A, i.e., there is at least one element in B which is not in A.

Example:-

(1) LetA={2,3,4}
B = {2, 3, 4, 5}

A is a proper subset of B.

(ii) The null Ø is a proper subset of every set.

(5) Improper Subset :- If A is a subset of B and A = B, then A is said to be an improper subset of B.

Example : (i) A = {2, 3, 4}, B = {2, 3, 4}

A is an improper subset of B.

(ii) Every set is an improper subset of itself.

(6) Universal Set :- If all the sets under investigations are subsets of a fixed set U, then the set U is called Universal Set.

Example :- In the human population studies the universal set consists of all the people in the world.

(7) Null Set or Empty Set :- A set having no elements is called a Null set or void set. It is denoted byø.

(8) Singleton Set :- It contains only one element. It is denoted by {s}.

Example :- S= $\{x|x \in N, 7< x <9\} = \{8\}$

(9) Equal Sets :- Two sets A and B are said to be equal and written as A = B if both have the same elements. Therefore, every element which belongs to A is also an element of the set B and every element which belongs to the set B is also an element of the set A.

$A = B \{x \in A \Leftrightarrow x \in B\}$.

If there is some element in set A that does not belong to set B or vice versa then $A \neq B$, i.e., A is not equal to B.

(10) Equivalent Sets :- If the cardinalities of two sets are equal, they are called equivalent sets. **Example**:- If A= {1, 2, 6} and B= {16, 17, 22}, they are equivalent as cardinality of A is equal to the cardinality of B. i.e. |A|=|B|=3

(11) Disjoint Sets :- Two sets A and B are said to be disjoint if no element of A is in B and no element of B is in A.

Example :- R={a,b,c}
S = {k, p, m}

R and S are disjoint sets.

(12) Power Sets :- The power of any given set A is the set of all subsets of A and is denoted by P (A). If A has n elements, then P (A) has 2^n elements.

Example :- A = {1, 2, 3}

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

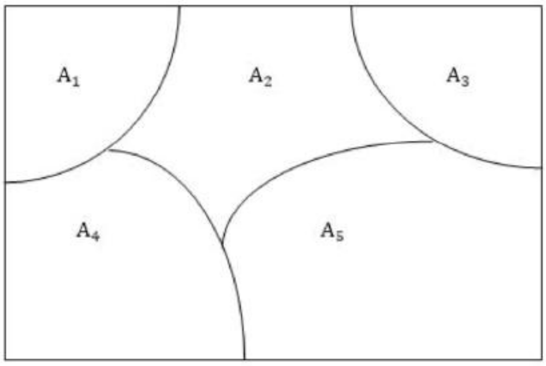
Partitions of a Set :- Let S be a nonempty set. A partition of S is a subdivision of S into nonoverlapping, nonempty subsets. Specifically, a partition of S is a collection {Ai} of nonempty subsets of S such that:

* Each a in S belongs to one of the Ai.
* The sets of {Ai} are mutually disjoint; that is,

$A_j \neq A_k$ Then $A_j \cap A_k = \emptyset$

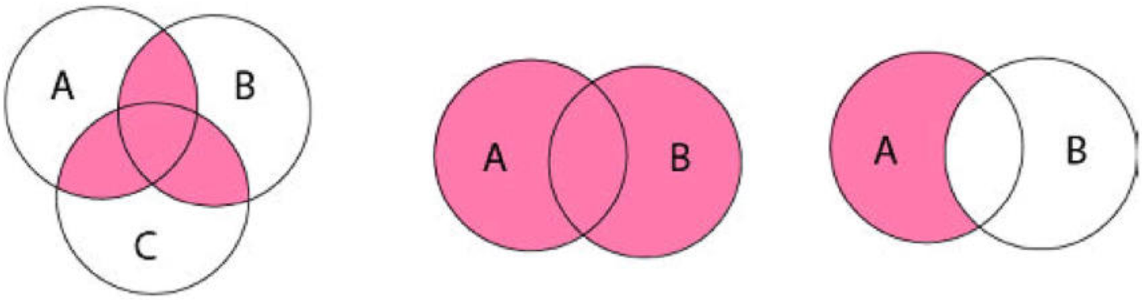
The subsets in a partition are called cells.

Fig:- Venn diagram of a partition of the rectangular set S of points into five cells, A_1, A_2, A_3, A_4, A_5



Venn Diagrams :- Venn diagram is a pictorial representation of sets in which an enclosed area in the plane represents sets.

Examples :-



Set Operations

Some commonly used set operations are:

Set Operation	**Description**	**Example**
Union ($\cup$)	The combination of elements from two sets, including duplicates.	if Set A = {1, 3, 5} and Set B = {2, 3, 4}, then $A \cup B$ = {1, 2, 3, 4, 5}
Intersection ($\cap$)	The collection of elements that are in both sets.	if Set A = {1, 3, 5} and Set B = {2, 3, 4}, then $A \cap B$ = {3}
Difference (\)	The collection of elements in the first set that are not in the second set.	if Set A {1, 3, 5} and Set B = {2, 3, 4}, then A\B = {1, 5}
Complement (A')	The collection of elements that are not in the first set, but are in the universal set (all elements under	if Universal Set = {1, 2, 3, 4, 5} and Set A = {2,4}, then A' = {1, 3, 5}

	consideration).	
Cartesian Product (A x B)	The collection of ordered pairs where the first element comes from the first set and the second element comes from the secondset.	if Set A = {1, 2} and Set B = {a, b}, then A x B = {(1, a), (1, b), (2, a), (2, b)}

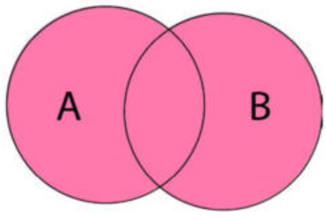
Operations on Sets :- The basic set operations are:

(1) Union of Sets :- Union of Sets A and B is defined to be the set of all those elements which belong to A or B or both and is denoted by AuB.

1. $\mathbf{A \cup B = \{x: x \in A \text{ or } x \in B\}}$

Example : Let A = {1, 2, 3}, B= {3, 4, 5, 6}

$A \cup B$ {1, 2, 3, 4, 5, 6}.

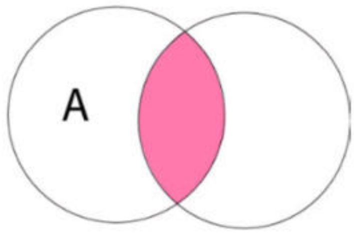


(2) Intersection of Sets :- Intersection of two sets A and B is the set of all those elements which belong to both A and B and is denoted by $A \cap B$.

1. $\mathbf{A \cap B = \{x: x = A \text{ and } x \in B\}}$

Example : Let A = {11, 12, 13}, B = {13, 14, 15}

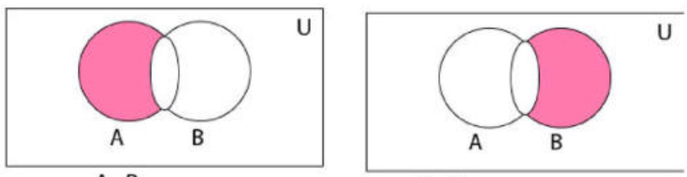
$A \cap B$ = {13}.



(3) Difference of Sets :- The difference of two sets A and B is a set of all those elements which belongs to A but do not belong to B and is denoted by A - B.

1. $\mathbf{A - B = \{x: x \in A \text{ and } x \notin B\}}$

Example : Let A = {1, 2, 3, 4} and B = {3, 4, 5, 6} then A - B = {3, 4} and B - A = {5, 6}



A - B B - A

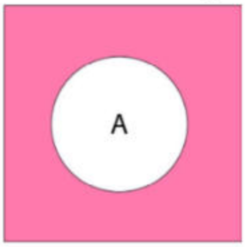
(4) Complement of a Set :- The Complement of a Set A is a set of all those elements of the universal set which do not belong to A and is denoted by A^c.

$A^c = U - A = \{x: x \in U \text{ and } x \notin A\} = \{x: x \notin A\}$

Example : Let U is the set of all natural numbers.

$A = \{1, 2, 3\}$

A^c = {all natural numbers except 1, 2, and 3}.



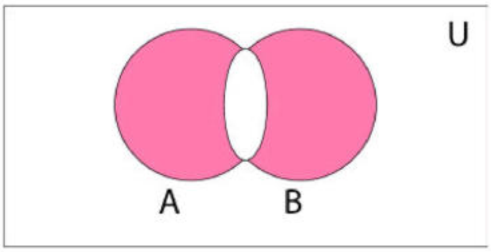
(5) Symmetric Difference of Sets : The symmetric difference of two sets A and B is the set containing all the elements that are in A or B but not in both and is denoted by $A \oplus B$ i.e.

1. **$A \oplus B$ $(A \cup B) - (A \cap B)$**

Example: Let A={a,b,c,d}

B={a,b,l,m}

$A \oplus B$ {c, d, I, m}



Algebra of Sets :- Sets under the operations of union, intersection, and complement satisfy various laws (identities) which are listed in Table 1.

Idempotent Laws	(a) $A \cup A = A$	(b) $A \cap A = A$
Associative Laws	(a) $(A \cup B) \cup C = A \cup (B \cup C)$	(b) $(A \cap B) \cap C = A \cap (B \cap C)$
Commutative Laws	(a) $A \cup B = B \cup A$	(b) $A \cap B = B \cap A$
Distributive Laws	(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(b) $A \cap (B \cup C) = (A \cap B)\ (A \cap C)$
De Morgan's Laws	(a) $(A \cup B)^c = A^c \cap B^c$	(b) $A \cap U = A$
Identity Laws	(a) $A \cup Ø = A$ (b) $A \cup U = U$	(c) $A \cap U = A$ (d) $A \cap Ø = Ø$

Complement Laws	(a) $A \cup A^c = U$	(b) $U^c = \emptyset$)
	(b) $A \cap A^c = \emptyset$	(d) $\emptyset^c = U$
Involution Law	(a) $(A^c)^c = A$	

Table : Law of Algebra of Sets

Table 1 shows the law of algebra of sets.

Example 1 :- Prove Idempotent Laws :-

(a) $A \cup A = A$

Solution: Since, $B \subset A \cup B$, therefore $A \subset A \cup A$

Let $X \in A \cup A \Rightarrow X \in A$ or $X \in A \Rightarrow X \in A$

$\therefore A \cup A \subset A$

As $A \cup A \subset A$ and $A \subset A \cup A \Rightarrow A = A \cup A$. Hence Proved.

(b) $A \cap A = A$

Solution: Since, $A \cap B \subset B$, therefore $A \cap A \subset A$

Let $X \in A \Rightarrow X \in A$ and $X \in A$

$\Rightarrow X \in A \cap A$ $\qquad\qquad \therefore A \subset A \cap A$

As $A \cap A \subset A$ and $A \subset A \cap A \Rightarrow A = A \cap A$. Hence Proved.

Example 2 : Prove Associative Laws:

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

Solution:

Let some $X \in (A' \cup B) \cup C$

$\Rightarrow (X \in A$ or $X \in B)$ or $X \in C$

$\Rightarrow X \in A$ or $X \in B$ or $X \in C$

$\Rightarrow X \in A$ or $(X \in B$ or $X \in C)$

$\Rightarrow X \in A$ or $X \in B \cup C$

$\Rightarrow X \in A \cup (B \cup C)$.

Similarly, if some $X \in A \cup (B \cup C)$, then $X \in (A \cup B) \cup C$.

Thus, any $\quad X \in A \cup (B \cup C) \Leftrightarrow X \in (A \cup B) \cup C$. Hence Proved.

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

Solution: Let some $X \in A \cap (B \cap C) \Rightarrow X \in A$ and $X \in B \cap C$

$\Rightarrow X \in A$ and $(X \in B$ and $X \in C) \Rightarrow X \in A$ and $X \in B$ and $X \in C$

$\Rightarrow (X \in A$ and $X \in B)$ and $X \in C) \Rightarrow X \in A \cap B$ and $X \in C$

$\Rightarrow X \in (A \cap B) \cap C$

Similarly, if some $X \in A \cap (B \cap C)$, then $X \in (A \cap B) \cap C$

Thus, any $\quad X \in (A \cap B) \cap C \Leftrightarrow X \in A \cap\subset (B \cap C)$.

Hence Proved.

Example 3 : Prove Commutative Laws

(a) $A \cup B = B \cup A$

Solution:

To Prove

$A \cup B = B \cup A$

$A \cup B = \{X : X \in A \text{ or } X \in B\}$

$= \{X : X \in B \text{ or } X \in A\}$ ($\therefore$ Order is not preserved in case of sets)

$A \cup B = B \cup A$. Hence Proved.

(b) $A \cap B = B \cap A$

Solution:

To Prove

$A \cap B = B \cap A$

$A \cap B = \{X : X \in A \text{ and } X \in B\}$

$= \{X : X \in B \text{ and } x \in A\}$ ($\therefore$ Order is not preserved in case of sets)

$A \cap B = B \cap A$. Hence Proved.

Example 4: Prove Distributive Laws

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

To Prove

Let $X \in A \cup (B \cap C) \Rightarrow X \in A \text{ or } X \in B \cap C$

$\Rightarrow (X \in A \text{ or } X \in A) \text{ or } (X \in B \text{ and } X \in C)$

$\Rightarrow (X \in A \text{ or } X \in B) \text{ and } (X \in A \text{ or } X \in C)$

$\Rightarrow X \in A \cup B \text{ and } X \in A \cup C$

$\Rightarrow X \in (A \cup B) \cap (A \cup C)$

Therefore, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ ......(i)

Again, Let $Y \in (A \cup B) \cap (A \cup C) \Rightarrow Y \in A \cup B \text{ and } Y \in A \cup C$

$\Rightarrow (Y \in A \text{ or } Y \in B) \text{ and } (Y \in A \text{ or } Y \in C)$

$\Rightarrow (Y \in A \text{ and } Y \in A) \text{ or } (Y \in B \text{ and } Y \in C)$

$\Rightarrow Y \in A \text{ or } Y \in B \cap C$

$\Rightarrow Y \in A \cup (B \cap C)$

Therefore, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$ .... (ii)

Combining (i) and (ii), we get $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Hence Proved

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution:

To Prove

Let $X \in A \cap (B \cup C) \Rightarrow X \in A$ and $X \in B \cup C$

$\Rightarrow (X \in A$ and $X \in A)$ and $(X \in B$ or $X \in C)$

$\Rightarrow (X \in A$ and $X \in B)$ or $(X \in A$ and $X \in C)$

$\Rightarrow X \in A \cap B$ or $X \in A \cap C$

$\Rightarrow X \in (A \cap B) \cup (A \cup C)$

Therefore, $A \cap (B \cup C) \subset (A \cap B) \cup (A \cup C)$ .... (i)

Again, Let $Y \in (A \cap B) \cup (A \cup C) \Rightarrow Y \in A \cap B$ or $Y \in A \cap C$

$\Rightarrow (Y \in A$ and $Y \in B)$ or $(Y \in A$ and $Y \in C)$

$\Rightarrow (Y \in A$ or $Y \in A)$ and $(Y \in B$ or $Y \in C)$

$\Rightarrow Y \in A$ and $Y \in B \cup C$

$\Rightarrow Y \in A \cap (B \cup C)$

Therefore, $(A \cap B) \cup (A \cup C) \subset A \cap (B \cup C)$....... (ii)

Combining (i) and (ii), we get $A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$. Hence Proved

Example 5: Prove De Morgan's Laws

(a) $(A \cup B)^c = A^c \cap B^c$

Solution: To Prove $(A \cup B)^c = A^c \cap B^c$

Let $X \notin (A \cup B) \Rightarrow X \notin A \cup B$ $(\therefore a \in a \Leftrightarrow a \notin A^c)$

$\Rightarrow X \notin A$ and $X \notin B$

$\Rightarrow X \notin A^c$ and $X \notin B^c$

$\Rightarrow X \notin A^c \cap B^c$

Therefore, $(A \cup B)^c \subset A^c \cap B^c$ ……………………..... (i)

Again, let $X \in A^c \cap B^c \Rightarrow X \in A^c$ and $X \in B^c$

$\Rightarrow X \notin A$ and $X \notin B$

$\Rightarrow X \notin A \cup B$

$\Rightarrow X \in (A \cup B)^c$

Therefore, $A^c \cap B^c \subset (A \cup B)^c$ ... (ii)

Combining (i) and (ii), we get $A^c \cap B^c = (A \cup B)^c$. Hence Proved.

(b) $(A \cap B)^c = A^c \cup B^c$

Solution:

Let $X \in (A \cap B) \Rightarrow X \notin A \cap B$ $(\therefore a \in A^c)$

$\Rightarrow X \notin A$ or $X \notin B$

$\Rightarrow X \in A^c$ and $X \in B^C$

$\Rightarrow X \in A^c \cup B^c$

$\therefore (A \cap B)^c \subset (A \cup B)$......... (i)

Again, Let $X \in A^c \cup B^c \Rightarrow X \in A^c$ or $X \in B^c$

$\Rightarrow X \notin A$ or $X \notin B$

$\Rightarrow X \notin A \cap B$

$\Rightarrow X \notin (A \cap B)^c$

$\therefore A^c \cup B^c \subset (A \cap B)^c$....................... (ii)

Combining (i) and (ii), we get $(A \cap B) = A^c \cup B^c$. Hence Proved.

Example 6: Prove Identity Laws.

(a) $A \cup \emptyset = A$

Solution: To Prove $A \cup \emptyset = A$

Let $X \in A \cup \emptyset \Rightarrow X \in A$ or $X \in \emptyset$

$\Rightarrow X \in A$ $\quad$ ($\therefore X \in \emptyset$, as $\emptyset$ is the null set)

Therefore, $X = A \cup \emptyset \Rightarrow X \in A$

Hence, $A \cup \emptyset \subset A$.

We know that $A \subset A \cup B$ for any set B.

But for $B = \emptyset$, we have $A \subset A \cap \emptyset$

From above, $A \subset A \cup \emptyset$, $A \cup \emptyset \subset A \Rightarrow A = A \cup \emptyset$. Hence Proved.

(b) $A \cap \emptyset = \emptyset$

Solution: To Prove $A \cap \emptyset = \emptyset$

If $X \in A$, then $X \notin \emptyset$ $\quad$ ($\because \emptyset$ is a null set)

Therefore, $X \in Am\ X \notin \emptyset \Rightarrow A \cap \emptyset = \emptyset$. Hence Proved.

(c) $A \cup U = U$

Solution :

To prove $A \cup U = U$

Every set is a subset of a universal set.

$\therefore A \cup U \subseteq U$

Also, $U \subseteq A \cup U$

Therefore, $A \cup U = U$. Hence Proved.

(d) $A \cap U = A$

Solution :

To prove $A \cap U = A$

We know $A \cap U \subset A$...................(i)

So we have to show that $A \subset A \cap U$

Let $X \in A \Rightarrow X \in A$ and $X \in U$ $\quad$ ($\because A \subset U$ So $X \in$

$A \Rightarrow X \in U)$

$\therefore X \in A \Rightarrow X \in A \cup U$

$\therefore A \subset A \cap U$ .... (ii)

From (i) and (ii), we get $A \cap U = A$. Hence Proved.

Example 7 : Prove Complement Laws

(a) $A \cup A^c = U$

Solution :

To prove $A \cup A^c = U$

Every set isa subset of U

$\therefore A \cup A^c \subset U$ ....(i)

We have to show that $U \subseteq A \cup A^c$

Let $X \in U \Rightarrow X \in A$ or $X \notin A$

$\Rightarrow X \in A$ or $X \in A^c \Rightarrow X \in A \cup A^c$

$\therefore U \subseteq A \cup A^c$ ....(ii)

From (i) and (ii), we get $A \cup A^c = U$. Hence proved.

(b) $A \cap A^c = Ø$

Solution :

As Ø is the subset of every set

$\therefore Ø \subseteq A \cap A^c$ ....(i)

We have to show that $A \cap A^c \subseteq Ø$

Let $X \in A \cap A^c \Rightarrow X$ A and $X \in A^c$

$\Rightarrow X \in A$ and $X \notin A$

$\Rightarrow X \in Ø$

$\therefore A \cap A^c \subset Ø$ ....(ii)

From (i) and (ii), we get $A \cap A^c = Ø$. Hence Proved.

(c) $U^c = Ø$

Solution :

Let $X \in U^c \Leftrightarrow X \notin U \Leftrightarrow X \notin Ø$

$\therefore U^C = Ø$ Hence Proved. (As U is the Universal Set).

(d) $Ø^C = U$

Solution :

Let $X \in Ø^c \Leftrightarrow X \notin Ø \Leftrightarrow X \in U$ (As Ø is an empty set)

$\therefore Ø^c = U$. Hence Proved.

Exampled 8 : Prove Involution Law

(a) $(A^c)^c$ A.

Solution :

Let $X \in (A^c)^c \Leftrightarrow X \notin A^c \Leftrightarrow X \in a$

$\therefore (A^c)^c = A$. Hence Proved.

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